Exercise 10

A particle moves with position function

$$s = t^4 - 4t^3 - 20t^2 + 20t \qquad t \geq 0$$

(a) At what time does the particle have a velocity of 20 m/s?

(b) At what time is the acceleration 0? What is the significance of this value of t?

Solution

Part (a)

To determine the velocity, take the derivative of the position function.

$$\begin{aligned} v(t) &= \frac{ds}{dt} \\ &= \frac{d}{dt}(t^4 - 4t^3 - 20t^2 + 20t) \\ &= 4t^3 - 12t^2 - 40t + 20 \end{aligned}$$

To find when the particle has a velocity of 20 m/s, set $v(t) = 20$ and solve the equation for t.

$$4t^3 - 12t^2 - 40t + 20 = 20$$

$$4t^3 - 12t^2 - 40t = 0$$

$$4t(t^2 - 3t - 10) = 0$$

$$4t(t-5)(t+2) = 0$$

$$t = \{-2, 0, 5\}$$

Therefore, the particle has a velocity of 20 m/s when $t = 0$ or $t = 5$.

Part (b)

Take the derivative of the velocity to get the acceleration function.

$$\begin{aligned} a(t) &= \frac{dv}{dt} \\ &= \frac{d}{dt}(4t^3 - 12t^2 - 40t + 20) \\ &= 12t^2 - 24t - 40 \end{aligned}$$

Set $a(t) = 0$ and solve for t to find when the acceleration is zero.

$$a(t) = 0$$

$$12t^2 - 24t - 40 = 0$$

$$4(3t^2 - 6t - 10) = 0$$

$$3t^2 - 6t - 10 = 0$$

$$t = \frac{6 \pm \sqrt{6^2 - 4(3)(-10)}}{2(3)}$$

$$t = \left\{ \frac{3 - \sqrt{39}}{3}, \frac{3 + \sqrt{39}}{3} \right\}$$

$$t \approx \{-1.08167, 3.08167\}$$

Since $t \geq 0$, choose the positive time.

$$t \approx 3.08167 \text{ s}$$

At this time the velocity is not changing; that is, the tangent line to the velocity curve is flat.